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Modelltheorie II

Homework Sheet 9

Deadline: 03.07.2023, 14 Uhr

Unless explicitly mentioned, we work inside a sufficiently large saturated model $\mathbb{U}$ of an ω-stable complete first-order theory T with infinite models in a fixed language $\mathcal{L}$.

Exercise 1 (4 Points). Let G be a definable group such that the formula "$x \in G$" is strongly minimal. Given a definable subgroup $H \leq G$, show that either H is finite or $H = G$.

Deduce from the above that $\mathbb{C}$ is both additively and multiplicatively connected.

Exercise 2 (7 Points). Let G be a group definable over the subset of parameters B of $\mathbb{U}$.

a) Show that every element of G can be written as a product of two elements, each generic over B.

b) Consider a generic element a over B. If $g \mathop{\smile\!\!\!\!\!\!|}\,_B a$, show that $g \cdot a$ is generic over B, g. In particular we have $g \cdot a \mathop{\smile\!\!\!\!\!\!|}\,_B g$.

c) Deduce that the product $g \cdot h$ of two elements g and h, both generic over B and with $g \mathop{\smile\!\!\!\!\!\!|}\,_B h$, is again generic over B. Moreover, show that $g \cdot h \mathop{\smile\!\!\!\!\!\!|}\,_B g$ and $g \cdot h \mathop{\smile\!\!\!\!\!\!|}\,_B h$.

d) Suppose now that an element a of G satisfies that $g \cdot a \mathop{\smile\!\!\!\!\!\!|}\,_B g$ whenever $g \mathop{\smile\!\!\!\!\!\!|}\,_B a$. Conclude that a is generic over B.

Exercise 3 (9 Points). Consider the saturated model $\mathbb{C}$ of the strongly minimal theory ACF_0 in the language of rings, as in Exercise 3 of the homework sheet 7.

a) Given $b_1, \ldots, b_n$ algebraically independent, let p be the type over $B = \{b_1, \ldots, b_n\}$ of maximal rank containing the definable set $X = \{(x_1, \ldots, x_n) \in \mathbb{C}^n \mid \sum_{i=1}^{n} b_i \cdot x_i = 1\}$. Describe explicitly the additive stabilizer $\mathrm{Stab}(p)$ of p, as a B-definable subgroup of $(\mathbb{C}^n, +)$. What is its Morley rank and degree?

Let now $Z = \{(x, y) \in \mathbb{C}^2 \mid y = x^2\}$ and q its unique type of maximal rank over $\emptyset$. Notice that q is stationary. Set $H = \mathrm{Stab}(q)$ and H^0 its connected component.

b) Given an element (g, h) of H^0 generic over a realization (a, b) of q, which algebraic equation does (g, h) satisfy over (a, b)?

c) Conclude that H^0 is trivial, using that H^0 is connected.

Hint: Take two suitable elements of H^0 over (a, b).

d) Is H trivial?

Die Übungsblätter können zu zweit eingereicht werden. Abgabe der Übungsblätter im Fach 3.07 im Keller des mathematischen Instituts.