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Übungen: Dr. Francesco Gallinaro — Sommersemester 2023

Modelltheorie II
Homework Sheet 7
Deadline: 19.06.2023, 14 Uhr

Exercise 1 (4 Points). Consider a sufficiently large cardinal $\kappa \geq \aleph_0$ and a strongly κ-homogeneous κ-saturated model $\mathbb{U}$ of a complete first-order theory T with infinite models in a fixed language $\mathcal{L}$. It follows from Exercise 4 in the homework sheet 6 that a type p in $\mathrm{S}(A)$ which is coheir over C (a subset of A) admits a global extension $\mathbb{p}$ over $\mathbb{U}$ which is coheir over C.

a) Show that $\mathrm{Cb}(\mathbb{p})$ is definable over C.

b) Is $\mathbb{p}$ definable over C?

Exercise 2 (10 Points). We work inside a sufficiently large saturated model $\mathbb{U}$ of an ω-stable complete first-order theory T with infinite models in a fixed language $\mathcal{L}$.

a) Given a set of parameters C and b in $\mathbb{U}$, show that $b \mathop{\smile\!\!\!\!\!\!\mid}_C b \iff b \in \mathrm{acl}(C)$.

We now consider a *independent sequence* $(a_n)_{n\in\mathbb{N}}$ *over* C, that is, $a_n \mathop{\smile\!\!\!\!\!\!\mid}_C a_0, \ldots, a_{n-1}$ for n in $\mathbb{N}$.

b) If $C \subset B$ with $B \mathop{\smile\!\!\!\!\!\!\mid}_C \{a_n\}_{n\in\mathbb{N}}$, show that $(a_n)_{n\in\mathbb{N}}$ is also an independent sequence over B.

c) Given indices $i_1, \ldots, i_n, j_1 \ldots, j_m$ with $\{i_1, \ldots, i_n\} \cap \{j_1, \ldots, j_m\} = \emptyset$, deduce from (a) that

$$\mathrm{acl}(C, a_{i_1}, \ldots, a_{i_n}) \cap \mathrm{acl}(C, a_{j_1}, \ldots, a_{j_m}) = \mathrm{acl}(C).$$

Hint: Every extension to the algebraic closure is non-forking.

d) Assume now that all a_n's have the same strong type over C. Show that $(a_n)_{n\in\mathbb{N}}$ is indiscernible over C.

Hint: Use stationarity to show inductively that $a_{i_1}, \ldots, a_{i_n} \equiv_C a_1, \ldots, a_n$ if $i_1 < \cdots < i_n$.

Exercise 3 (6 Points). We work inside a sufficiently large saturated model $\mathbb{U}$ of an ω-stable complete first-order theory T with infinite models in a fixed language $\mathcal{L}$.

a) Show that an instance $\varphi[x, a]$ with a in $\mathbb{U}$ has Morley degree 1 if and only if there is a unique type of maximal rank containing $\varphi[x, a]$ over each set B containing a.

Consider the saturated model $\mathbb{C}$ of the strongly minimal theory ACF_0 in the language of rings.

b) Given $b_1, \ldots, b_n$ algebraically independent, show that the definable set

$$X = \left\{ (x_1, \ldots, x_n) \in \mathbb{C}^n \mid \sum_{i=1}^{n} b_i \cdot x_i = 1 \right\}$$

has Morley degree 1. What is its Morley rank?

c) Let $\mathbb{p}$ be the unique global type containing the definable set X. Show that $\mathrm{Cb}(\mathbb{p})$ is interdefinable with $(b_1, \ldots, b_n)$.

Die Übungsblätter können zu zweit eingereicht werden. Abgabe der Übungsblätter im Fach 3.07 im Keller des mathematischen Instituts.