# Modelltheorie II

Homework Sheet 5

Deadline: 05.06.2023, 14 Uhr

Throughout, we consider some sufficiently large cardinal $\kappa \geq \aleph_0$ and a strongly $\kappa$-homogeneous $\kappa$-saturated model $\mathbb{U}$ of a complete first-order theory $T$ with infinite models in a fixed language $\mathcal{L}$.

**Exercise 1** (5 Points). Assume that for every definable subset $X$ in $\mathbb{U}$ there exists a smallest algebraically closed subset $A$ of parameters over which $X$ is definable.

Show that $T$ has weak elimination of imaginaries.

**Hint:** What is the cardinality of $\mathrm{acl}(A_0)$ for $A_0$ finite?

**Exercise 2** (5 Points). Given an interpretable equivalence relation $F$ without parameters on some cartesian product of imaginary sorts, show that there is 0-definable in $\mathcal{L}$ equivalence relation $E$ such that every class modulo $F$ is interdefinable with an imaginary in the sort $S_E$.

Deduce that $T^{eq}$ has uniform elimination of imaginaries.

**Exercise 3** (10 Points). Consider an indiscernible sequence $(a_n)_{n\in\mathbb{Z}}$ of tuples of some fixed length. Given two non-empty finite subsets $I$ and $J$ of $\mathbb{Z}$, we say that $I \ll J$ if

$$\max I < \min J.$$

a) Show that the kernel of the sequence

$$K = \bigcup_{\substack{I,J \underset{\text{finite}}{\subset} \mathbb{Z} \\ I \ll J}} \mathrm{acl}^{\mathrm{eq}}(\{a_i\}_{i\in I}) \cap \mathrm{acl}^{\mathrm{eq}}(\{a_j\}_{j\in J})$$

equals $\mathrm{acl}^{\mathrm{eq}}(\{a_n\}_{n<0}) \cap \mathrm{acl}^{\mathrm{eq}}(\{a_m\}_{m>0})$.

**Hint:** Automorphisms.

b) Show that the kernel is the largest subset of $\mathrm{acl}^{\mathrm{eq}}(\{a_n\}_{n\in\mathbb{Z}})$ over which the sequence is indiscernible.

**Hint:** Every sequence of tuples of the original $a_n$'s, ordered increasingly, is again indiscernible.

We now assume that $T$ is the theory of infinite sets in the empty language.

c) Construct a non-constant indiscernible sequence, each member consisting of a pair of real elements, whose kernel is not trivial, that is, the kernel is not contained in $\mathrm{acl}^{\mathrm{eq}}(\emptyset)$.

d) In the sequence $\{a_n\}_{n\in\mathbb{Z}}$ obtained in $c$), does $\mathrm{tp}(a_1/K, a_0)$ fork over $K$?

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Die Übungsblätter können zu zweit eingereicht werden. Abgabe der Übungsblätter im Fach 3.07 im Keller des mathematischen Instituts.